INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2014-15
Statistics - II, Backpaper Examination

1. The life time X, of a certain product, has the $Exponential(\theta)$ distribution with density
$f(x|\theta) = \theta \exp(-\theta x)$, $x > 0$, $\theta > 0$.
Let $X_1, \ldots, X_n$ be life times of a random sample of $n > 1$ such products. Consider testing

$$H_0 : \theta \leq \theta_0 \text{ versus } H_1 : \theta > \theta_0.$$

(a) Show that the conditions required for the existence of UMP test are satisfied here.
(b) Derive the UMP test of level α. **[10]**

2. Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ be independent random samples, respectively, from $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, where μ_1, μ_2 and σ^2 are unknown. For testing

$$H_0 : \mu_1 = \mu_2 \text{ versus } H_1 : \mu_1 \neq \mu_2,$$

find the generalized likelihood ratio test at the significance level α. **[10]**

3. The number of occurrences of a certain disease, X, is assumed to have the Poisson(λ) distribution with mean λ. Consider data $X_1, \ldots, X_n$ from $n > 1$ widely separated areas.
(a) Derive the maximum likelihood estimator, $\hat{\lambda}$, of λ.
(b) Is $\hat{\lambda}$ a consistent estimator of λ?
(c) Derive the asymptotic distribution of $\hat{\lambda}$.
(d) Derive the asymptotic distribution of $\left(\hat{\lambda}\right)^{1/2}$.
(e) Find a large sample 95% confidence interval for λ using (d). **[15]**

4. Consider a trial which ends up in 'Success' with probability p or 'Failure' with probability $1 - p$, $0 < p < 1$. Let X denote the number of independent trials required to obtain the first 'Success'. Let $X_1, \ldots, X_n$ be a random sample from the distribution of X. Assume the Beta(a, b) prior distribution on p.
(a) Derive the posterior distribution p given the data.
(b) Find the highest posterior density estimate of p.
(c) Find the posterior mean and posterior standard deviation of p. **[15]**